1997

Seeking the Profitability-Risk-Competitiveness Frontier Using a Genetic Algorithm

Ronnie Tan
Equitable of Iowa Companies, rytan@equitable-of-iowa.com

Journal of Actuarial Practice Vol. 5, No. 1, 1997

Seeking the Profitability-Risk-Competitiveness Frontier Using a Genetic Algorithm

Ronnie Tan*

Abstract

Monte Carlo simulation is used to develop a flexible framework to measure the profitability, risk, and competitiveness of any insurance product. A genetic algorithm is then used to seek the optimum asset allocations that form the profitability-risk-competitiveness frontier and to examine the profitability, risk, and competitiveness trade-offs. We also show how to select the appropriate asset allocation and crediting strategy in order to position the product at the desired location on the profitability-risk-competitiveness spectrum.

Key words and phrases: *asset allocation, product positioning, risk-based capital, Monte Carlo simulation, capital asset pricing model*

1 Introduction

Monte Carlo[1] simulation has been widely accepted as a valid method of valuing path-dependent cash flows. Hull (1993) applies Monte Carlo simulation techniques to derivatives, Hayre and Lauterbach (1995) apply them to mortgage-backed securities, and Asay, Bouyoucos, and Marciano (1993) apply them to pricing single premium deferred annuities. In recent years, asset-liability management actuaries also have begun

*Ronnie Tan, F.S.A., M.A.A.A., is an actuarial assistant in the Corporate Finance department at Equitable of Iowa Companies. He received his B.S. degree in actuarial science from the University of Nebraska-Lincoln in 1994. Since joining Equitable of Iowa Companies in 1994, his assignments have included interest rate risk management, equity index annuity hedging, dynamic pricing, corporate financial projections, cash flow testing, risk based capital, and surplus management.

Mr. Tan's address is: Equitable of Iowa Companies, 909 Locust Street, Des Moines, IA 50309-2899, USA. Internet address: *rytan@equitable-of-iowa.com*

[1]For an introduction to Monte Carlo simulation in general see, for example, Kalos and Whitlock (1986).

using Monte Carlo simulation to value insurance company liabilities and these liabilities' option-adjusted durations and convexities.[2]

Although asset-liability management actuaries are using Monte Carlo simulation in their work, product development actuaries have yet to widely apply it. This is because Monte Carlo simulation techniques, as they now exist, do not address the needs of the product development actuary. In pricing their products, product development actuaries want to know the following:

- Will the capital invested by the company earn more than the company's cost of capital?
- Will the risk of selling the product be acceptable?
- Will the product be competitive enough to meet the company's sales target?
- How should the company invest in order to balance the above three criteria?

The 1993 paper by Asay, Bouyoucos, and Marciano does not meet the above needs for product development actuaries for the following reasons:

- Product development actuaries are concerned with distributable earnings after considering the surplus required to satisfy both regulators and rating agencies. They are concerned with the liability cash flows only when they impact distributable earnings.
- Profitability, risk, and competitiveness of a product must be considered in pricing. By matching durations and convexities, an attempt is made to minimize the risk component of the product. It may be desirable, however, to take additional risk to enhance the profitability and competitiveness of a product. Asay, Bouyoucos, and Marciano formulated a profitability measure using option-adjusted spreads. Their measure is, however, deficient because it only requires an asset option-adjusted spread that is greater than the liability option-adjusted spread. Though such option-adjusted spreads may result in positive profits, they do not ensure that the profits are sufficient to cover the cost of capital.

[2]For more discussion and detail on the market value of liabilities, see the American Academy of Actuaries Task Force on Fair Valuation of Liabilities (1995), Merfeld (1996), and Reitano (1997).

- The investment allocation likewise must be considered in pricing. While asset-liability management actuaries extensively work to determine the ideal asset allocation to back the liability, their definition of "ideal" has always been that of minimizing risk. A new definition that balances profitability, risk, and competitiveness is needed.

Section 2 introduces a framework through which profitability, risk, and competitiveness of a product can be measured. In calculating profitability and risk, emphasis is placed on distributable earnings.[3] These earnings are not only determined by the profit stream of the product, but are also dependent on the target surplus required by regulators and rating agencies. We use the National Association of Insurance Commissioners (NAIC) risk-based capital formula as a proxy for the target surplus needed to be held.

In Section 3 we provide a brief description of a genetic algorithm and provide references for more detailed information. A genetic algorithm is then used to determine the ideal asset allocation for a single premium deferred annuity. We show that there is no single asset allocation that can be considered ideal. Rather, we have a set of asset allocations forming the profitability-risk-competitiveness frontier. With the genetic algorithm, asset allocations close to the frontier can be found.

Section 4 describes a method to quantify the profitability-risk-competitiveness trade-offs. Once these factors are quantified, decisions on where to position the product on the profitability-risk-competitiveness spectrum can be made by restating the first three concerns of product development actuaries as constraints on each of the three measures defined in Section 2.

2 Profitability-Risk-Competitiveness Framework

For the n-th path of the Monte Carlo simulation, let $r_{y,t,n}$ denote the yield curve path with $y = 0.25, 0.5, 1, 2, \ldots, 30$ denoting the point on the yield curve, $t = 0, 1, 2, \ldots, T$ denoting the time period (be it monthly, quarterly, or annually), and $n = 1, 2, \ldots, N$ denoting the path number. Many models can be used to generate these paths. Under each model, many decisions must be made about the values to be used for parameters. It is beyond the scope of this paper to describe and

[3] Distributable earnings are after tax statutory earnings plus changes in target surplus, i.e., they are net cash flows facing the insurance company. They represent how much the insurance company has to put in and how much it can take out from the project (of selling the product).

evaluate the various models available. The model selected should be arbitrage free, however, and the values used for the parameters in the model should be justifiable. Models that are not arbitrage free will result in mispricing the various asset classes. Hull (1993) describes some of these models.

Once the yield curve paths have been generated, an asset-liability model is used to calculate the distributable earnings of the insurance product. The asset-liability model will consider the asset allocation used, the investment strategy under positive and negative cash flows, the interest-crediting strategy of the product, the required target surplus, competitor rates, and mortality, lapse, and expense assumptions. It is beyond the scope of this paper to detail asset-liability modeling. Asay, Bouyoucos, and Marciano (1993) provide further detail on the topic.

Let $DE_{t,n}$ denote the distributable earnings for time period t and the path number n. The present value of distributable earnings for path n can be calculated as

$$PVDE_n = \sum_{t=0}^{T} \frac{DE_{t,n}}{\prod_{k=0}^{t-1}(1 + r_{0.25,k,n} + s)^{\Delta t}} \tag{1}$$

where Δt is the length of each time period and s is the risk premium that must be added to the risk free rate in order to arrive at the appropriate discount rate for distributable earnings.

From the standard capital asset pricing model (CAPM), the required return on a project or a security, R, is given by

$$R = R_F + \beta(R_M - R_F)$$

where R_F is the risk free rate, β is a measure of the riskiness of the project or security relative to the market, and $R_M - R_F$ is the market risk premium. If we assume that the product being priced has risk similar to that of the insurance company as a whole, and the β of the insurance company can be reasonably estimated, the risk premium in equation (1) can be replaced with:

$$s = \beta(R_M - R_F). \tag{2}$$

The market risk premium $R_M - R_F$ has been estimated by Brealey and Myers (1991, Chapter 8, page 161) to be 8.40 percent, resulting in $s = 0.084\beta$. Clearly s is similar to the option-adjusted spread of mortgage-backed securities as described in Hayre and Lauterbach (1995) and the option-adjusted spread of single premium deferred annuities as described in Asay, Bouyoucos, and Marciano (1993).

The profitability measure now can be defined as the sample mean of *PVDE* (i.e., $\hat{E}[PVDE]$) and the risk measure as its sample standard deviation (i.e., $STD[PVDE]$), i.e.,

$$\text{Profitability Measure} = \frac{1}{N}\sum_{n=1}^{N} PVDE_n \tag{3}$$

$$\text{Risk Measure} = \sqrt{\frac{1}{N-1}\sum_{n=1}^{N}(PVDE_n - E[PVDE])^2}. \tag{4}$$

Finally, the competitiveness measure can be tailored to the type of product. It may be appropriate to look at the credited interest rate for deferred annuities and fund-based life insurance such as universal life. For traditional life insurance we may want to use the annual premium as the competitiveness criterion, while for immediate annuities we may want to examine the periodic payments the policyholder will receive.

3 Genetic Algorithms: Overview and Example

3.1 What is a Genetic Algorithm?[4]

The birth of the genetic algorithm is generally attributed to Holland (1975). Descriptions of the ideas behind these algorithms can be found in Goldberg (1989).

Genetic algorithms are fundamentally different from classical algorithms. The differences are based on four principles (Goldberg, 1989):

1. Genetic algorithms use a coded representation of the parameters, not the parameters themselves;

2. Genetic algorithms search from a population of solution vectors, not a single solution vector;

[4]The description of a genetic algorithm used in this section is taken from Brigger (1995). Brigger's dissertation can be accessed on the World Wide Web at *http://ltswww.epfl.ch/pub_files/brigger/thesis_html/node58.html.* One can also visit genetic algorithm websites such as *http://www.aic.nrl.navy.mil/galist/* for more information. Applications of genetic algorithms to actuarial investments are given in Barber (1995) and Wendt (1995).

3. Genetic algorithms exclusively use values of the function under study and do not consider auxiliary information, such as the derivative; and

4. Genetic algorithms use probabilistic transition rules, not deterministic rules.

The function parameters—or the "living being"—are represented by a structure called a chromosome. Genetic algorithms manipulate chromosomes to profit from and exploit similarities between different performing chromosomes. Genetic algorithms optimize a population of chromosomes, unlike other methods that optimize only a single solution vector. The probability to select a false solution is reduced by considering several solution vectors of high performance. Genetic algorithms remain highly general because their optimization is directly based on the function values. There are no limitations set to continuous and derivable functions. Transition rules of genetic algorithms are stochastic and not deterministic as in many other algorithms. Yet there remains an important difference between genetic algorithms and random search algorithms, where decisions uniquely based on pure chance guide the exploration. Genetic algorithms benefit largely from the available information within the current population and use chance only to guide their exploration.

As in any optimization procedure, three associated objects are characteristic for genetic algorithms:

1. The environment of the system undergoing optimization;

2. The adaptive plan that determines successive structural modifications in response to the environment; and

3. A measure of the performance of different chromosomes in the environment.

The first and third points are given by the problem, and the task of the genetic algorithm is to control the mixture of operators that affect the system undergoing optimization. Thus, the workings of the system are conveyed in the adaptive plan which determines what chromosomes arise in response to its environment. A given chromosome performs differently in different environments, it is more or less fit, and it is the adaptive plan's task to produce chromosomes that perform well (are fit) in the environment confronting it.

The key problem for the adaptive plan is that it has absolutely no information about which chromosomes are most fit. In order to obtain this information, the plan must test and evaluate different chromosomes within the environment. Based on the fitness of each, the adaptive plan draws a selection. The adaptiveness is invoked when different environments cause different chromosomes to be selected. Successive structural modifications dictated by an adaptive plan amount to a sequence or trajectory through the set of all attainable chromosomes. For a plan to be adaptive, the trajectory must depend upon which environment is present.

3.2 An Example Using a Deterministic Genetic Algorithm

Suppose we have a single premium deferred annuity (SPDA) as described in Table 1 and an asset-liability model with assumptions as described in Table 2. Furthermore, we have the ability to invest in the types of assets as shown in Table 3.

Table 1
Description of SPDA

Policy Characteristics	Description
Premium:	$1,000,000;
Average Policy Size:	$50,000;
Guarantee Period:	1 year for both initial and renewal rates;
Minimum Rate:	3.00%;
Surrender Charges:	7% year 1, declining 1% per year;
Free Withdrawal:	10%;
Sales Commissions:	5% of premium;
Issuance Expenses:	0.1% of premium;
Administrative Expenses:	$20 per policy per annum;
Death Benefit:	Fund value.

Given the specifications of the SPDA and the asset-liability model, the goal is to determine the asset allocation that will satisfy the product development actuary in terms of profitability, risk, and competitiveness. With ten different types of assets, however, there is a wide range of possible asset allocations to consider. This is where a genetic algorithm is able to help. Genetic algorithms allow solutions to evolve from one generation to the next, with the new generation of solutions superior to the prior generation.

Table 2
Asset-Liability Model Assumptions

Model Component	Assumption
Projection Period:	25 years;
Target Surplus:	200% of NAIC company action level capital;
Federal Income Tax Rate:	35%;
Company Beta:	1.00;
Mortality:	1975-1980 SOA mortality rate;
Sex Distribution:	50% male, 50% female;
Issue Age:	60;
Base Lapses:	2%, 3%, 4%, 5%, 6%, 7%, 8%, 30%, 10%, 10% ...;
Dynamic Lapses:	Considers difference between competitor rate and credited rate, and surrender charge remaining;
Maximum Lapses:	50%;
Competitor Rate:	Maximum of five year treasury rate and 3.5%;
Crediting Strategy:	Subtract 220 basis points from asset yield;
Investment Strategies Under	
—Positive Cash Flows:	Invest in the same allocation as the initial allocation;
—Negative Cash Flows:	Sell assets in proportion to their market values at the time of sale;
Number of Scenarios:	100, starting with the December 31, 1996 yield curve.

To begin the genetic algorithm process, it is necessary to define the first generation of solutions. One possible way is to define asset allocations that are 100 percent of each of the asset types. Table 4 shows the results of these allocations. We observe the following from Table 4:

- The level of profitability is higher for A bonds, less for commercial mortgages, and least for BB bonds. Based on the NAIC risk-based capital formula, A bonds require the least capital, commercial mortgages more capital, and BB bonds the most capital.
- Investing in longer maturity assets will result in higher risk. In general, the market values of longer maturity assets are more sensitive to changes in the yield curve. Because policyholders have the option to surrender their policies at book value, the insurance company is exposed to most of the fluctuations in the market values of the assets.
- Because of the crediting strategy of deducting a fixed spread from the asset yield to arrive at the credited rate, assets with higher yields will result in a more competitive SPDA. There is no requirement that the spread has to be the same for different asset allocations.

Table 3
Types of Assets Available to Invest

Type	Asset	Spread Over Treasury Curve
1	3 Year Noncallable A Bond	+35 basis points
2	5 Year Noncallable A Bond	+40 basis points
3	7 Year Noncallable A Bond	+45 basis points
4	10 Year Noncallable A Bond	+60 basis points
5	5 Year Noncallable BB Bond	+165 basis points
6	7 Year Noncallable BB Bond	+185 basis points
7	10 Year Noncallable BB Bond	+220 basis points
8	5 Year Commercial Mortgage	+140 basis points
9	7 Year Commercial Mortgage	+145 basis points
10	10 Year Commercial Mortgage	+150 basis points

Note: All commercial mortgages have a 20 year amortization schedule.

Table 4
Generation 1 Results

Asset Allocation: 100% in	Mean	STD	ICR
3 Year Noncallable A Bond	15,654	7,657	4.13%
5 Year Noncallable A Bond	11,346	13,184	4.35%
7 Year Noncallable A Bond	6,391	18,787	4.55%
10 Year Noncallable A Bond	3,377	25,371	4.79%
5 Year Noncallable BB Bond	-11,018	11,340	5.25%
7 Year Noncallable BB Bond	-17,287	16,700	5.59%
10 Year Noncallable BB Bond	-21,132	22,415	6.04%
5 Year Commercial Mortgage	-33	8,765	5.48%
7 Year Commercial Mortgage	-1,496	12,797	5.67%
10 Year Commercial Mortgage	-1,074	17,445	5.81%

Notes: MEAN = Mean of *PVDE*; STD = Standard Deviation of *PVDE*; and ICR = Initial Credited Rate.

To create the second generation of solutions, it is possible to select any two first generation solutions and combine them. Thus, combining a 100% Type 2 (i.e, in 5-year-noncallable-A-bonds) allocation and a 100% Type 8 (i.e., in 5-year-commercial-mortgages) allocation will result in an asset allocation with a 50 percent weight in each of Types 2 and 8 assets. With ten different allocations from which to choose, we are able to create 45 (i.e., $\binom{10}{2}$) new asset allocations to form the second generation of solutions. After the second generation, with 55 different asset allocations in our solution set we can create up to 1,485 (i.e., $\binom{55}{2}$) different asset allocations to form the third generation of solutions. It is impractical, however, to run the asset-liability model for so many different asset allocations. Biased reproduction, based on the fitness of an asset allocation, is imposed to limit the number of solutions we have for the third and later generations.

Let Ω denote the set of all asset allocations that can be formed from the ten asset types shown in Table 3. For an asset allocation $j \in \Omega$, let $F(j)$ denote its fitness score, defined as:

$$F(j) = F^{(P)}(j) + F^{(R)}(j) + F^{(C)}(j) \tag{5}$$

where

$$F^{(P)}(j) = \frac{E[PVDE]_j - E[PVDE]_{\min}}{E[PVDE]_{\max} - E[PVDE]_{\min}} \quad (6)$$

$$F^{(R)}(j) = \frac{STD[PVDE]_{\max} - STD[PVDE]_j}{STD[PVDE]_{\max} - STD[PVDE]_{\min}} \quad (7)$$

$$F^{(C)}(j) = \frac{ICR_j - ICR_{\min}}{ICR_{\max} - ICR_{\min}} \quad (8)$$

$E[PVDE]_j$ = Mean of $PVDE$ for allocation j

$STD[PVDE]_j$ = Standard deviation of $PVDE$ for allocation j, and

ICR_j = Initial credited rate for allocation j.

From Table 4 we obtain:

$$\begin{aligned} E[PVDE]_{\max} &= 15{,}654 \\ E[PVDE]_{\min} &= -21{,}132 \\ STD[PVDE]_{\max} &= 25{,}371 \\ STD[PVDE]_{\min} &= 7{,}657 \\ ICR_{\max} &= 6.04\% \\ ICR_{\min} &= 4.13\%. \end{aligned}$$

If there are two asset allocations $j, k \in \Omega$ such that $F^{(P)}(j) < F^{(P)}(k)$, $F^{(R)}(j) < F^{(R)}(k)$, and $F^{(C)}(j) < F^{(C)}(k)$, we say that asset allocation k dominates asset allocation j. When this occurs we force asset allocation j to become extinct and eliminate it from our solution set. Asset allocations that are not dominated by other asset allocations will remain in the solution set. Only the fittest asset allocations (i.e., those with the highest fitness scores) of these survivors, however, will combine to produce new asset allocations for the next generation.

A conscious effort is made to ensure that solutions across the profitability-risk-competitiveness spectrum are achieved so that the final solution set is not concentrated in a small part of the spectrum. Also, for diversity purposes, asset allocations representing as many different asset types as possible are selected to reproduce. After six generations, it can be observed that there is no longer much improvement in the fitness scores, implying that the solution set is close to the profitability-risk-competitiveness frontier.

Tables A1 through A8 in the appendix show all the asset allocations in our solution set after generation six, while Figures 1 and 2 show the evolution of our solution set from generation one to generation six. Figures 1 and 2 are similar to the familiar efficient frontier of portfolio theory, except they contain an additional variable. Instead of an efficient frontier in the profitability-risk space, we now have an efficient frontier in the profitability-risk-competitiveness space.

Figures 1 and 2 show that there is no asset allocation that simultaneously yields the highest profitability, the least risk, and the highest credited rate. In order to achieve a higher credited rate for the SPDA, we must reduce the profitability or increase the risk. We will now explore these trade-offs.

4 Profitability-Risk-Competitiveness Trade-Offs

Figure 1 does not allow us to quantify the trade-off between risk and competitiveness because the profitability measure has not been held constant. Similarly, Figure 2 does not allow us to quantify the trade-off between profitability and competitiveness because the risk measure has not been held constant. To view the various trade-offs, we must include all three measures in a single graph. Before we construct such a graph, two points must be made:

- Not all of the asset allocations shown in the appendix are feasible. Although the profitability-risk-competitiveness profiles of these allocations are sufficiently attractive to allow them to survive through all generations, some of these allocations may not be in line with the investment policy of the insurance company. One such policy may be the requirement that the asset allocation cannot be too concentrated in any one asset category to reduce credit and sector risks. Because the model used in this paper does not consider such risks, there is a tendency for these risks to be ignored.

- We have assumed that the credited rate is determined by deducting 220 basis points from the asset yield. The spread of 220 basis points is by no means magical, and the insurance company is not bound to that number. The insurance company may even use a different spread for each policy year. We now assume that the spread can be any number. To simplify matters, however, we continue to assume that a level spread is used. For any given asset allocation, changing the spread will change the profitability-risk-competitiveness profile of that allocation.

Figure 1
Initial Credited Rate Versus *STD*[*PVDE*]

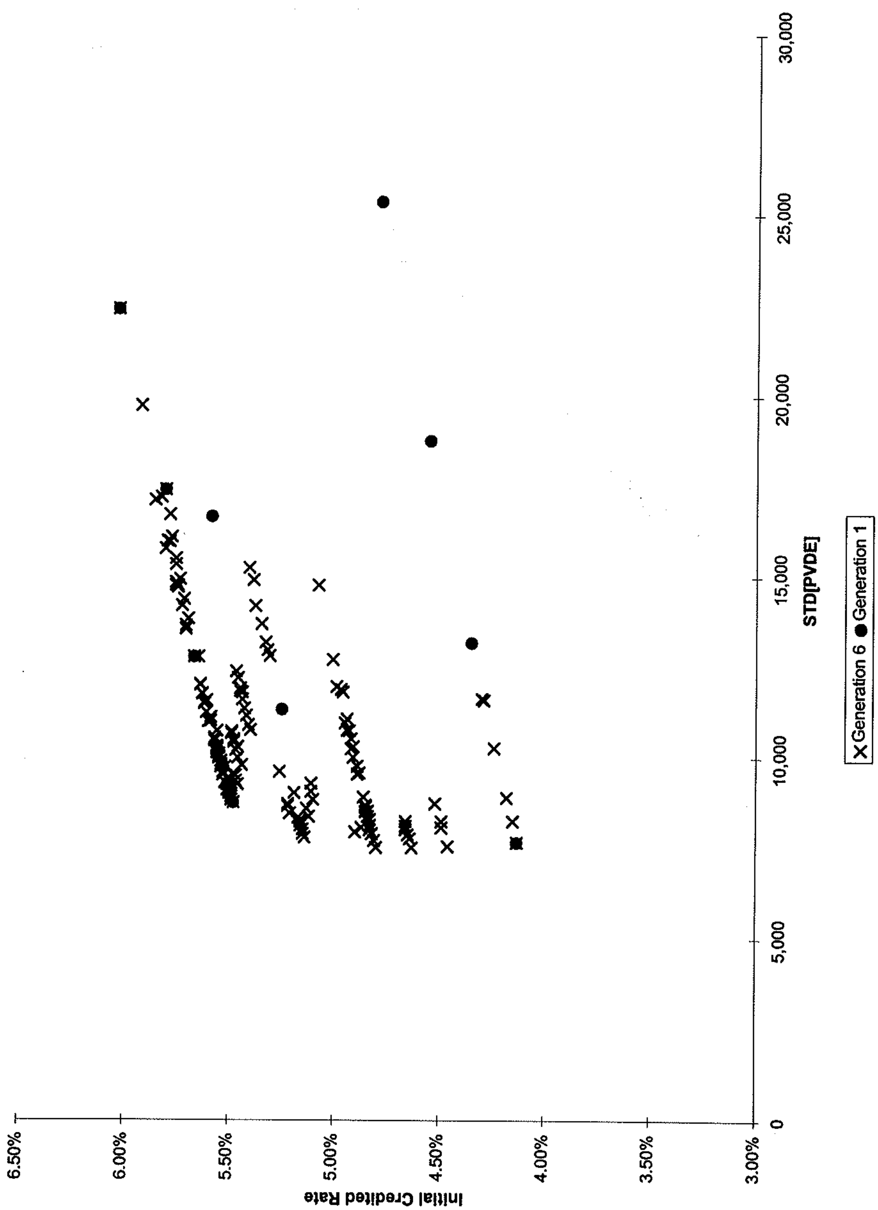


Figure 2
Initial Credited Rate Versus $E[PVDE]$

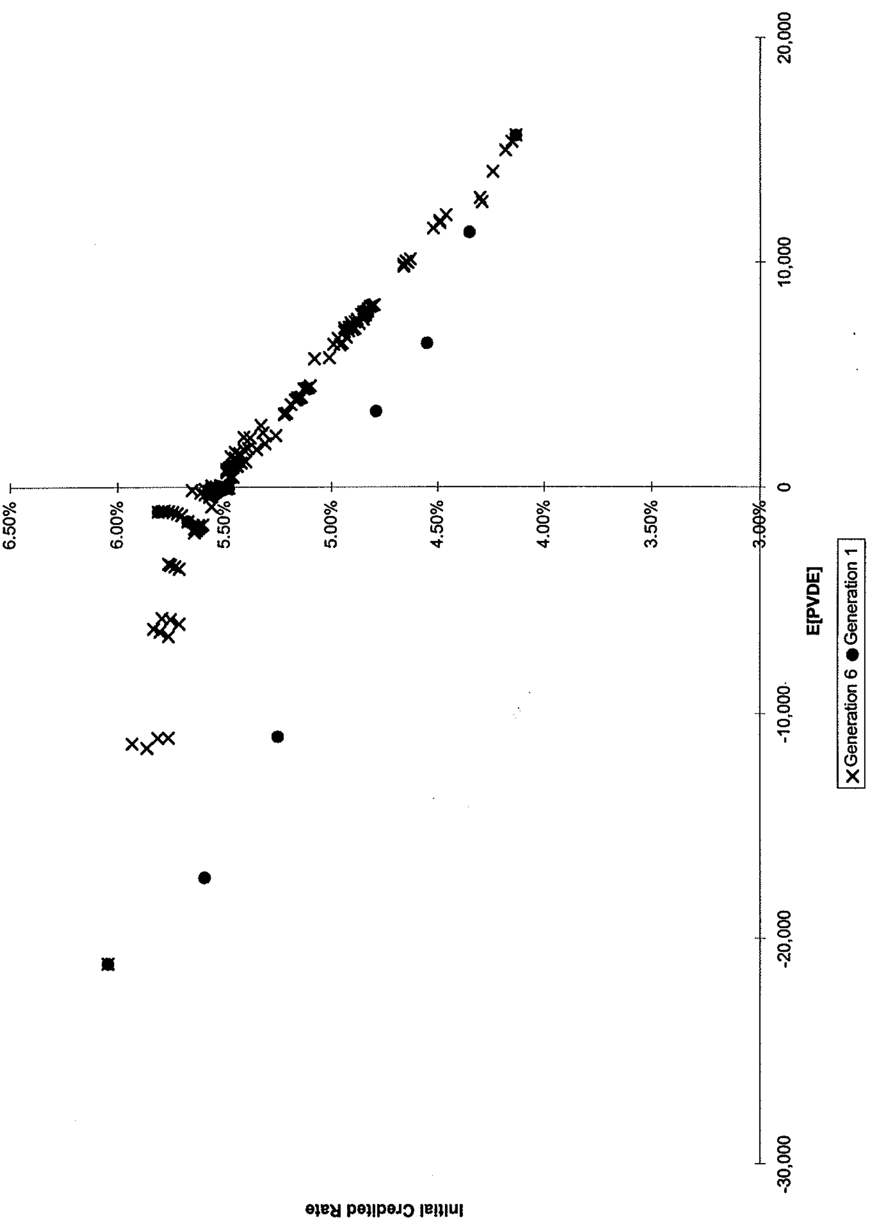


Figure 3 redraws the final solution set shown in Figure 2 with two changes. First, the policy of limiting the combined allocation of BB bonds and commercial mortgages to 50 percent of the asset portfolio is imposed. Second, the spread of 220 basis points is relaxed to range from 20 basis points to 420 basis points.

In Figure 3, points A, B, C, and the four points between them belong to the same asset allocation, with spreads ranging from 20 basis points to 420 basis points. The same case applies to points D, E, F, and the four points between them. Limiting our choice to asset allocations having a combined BB bonds and commercial mortgages allocation of not more than 50 percent reduces our solution set to the line BE while allowing the ability to change the spread to something other than 220 basis points expands our solution set to the area inside ABCFED. Any point within the area ABCFED can be reached by selecting an asset allocation along the line BE and then changing the spread.

Figure 3 shows that when the spread is reduced to increase the credited rate, profitability is adversely impacted. Moving from point B to point A increases the credited rate by 200 basis points from 5.08 percent to 7.08 percent, but reduces $E[PVDE]$ from 5,695 to -55,876. Risk is reduced as well when the credited rate is increased. As shown in Figure 4, $STD[PVDE]$ is reduced from 14,783 at point B to 4,690 at point A. Crediting a high rate will make policyholders less likely to surrender their policies in rising interest rate scenarios, thereby reducing the loss from the sale of assets at low market values and making profits more stable.

From the $STD[PVDE]$ values shown on Figure 4, it is possible to draw rough equivalent risk curves representing the points on the profitability-competitiveness space where the risk level is the same. As expected, these curves have negative slopes which implies that if we keep the risk level constant, the only way to increase profitability is to reduce credited rate. The only way to increase credited rate is to reduce profitability. Also, by increasing the acceptable risk level (shifting to a higher equivalent risk curve), we are able to increase profitability while keeping credited rate the same, increase the credited rate while keeping profitability the same, or even increase both profitability and the credited rate.

Figure 4 allows us to measure the various profitability-risk-competitiveness trade-offs. The trade-off between profitability and competitiveness can be obtained by moving along an equivalent risk curve; the trade-off between profitability and risk can be obtained by moving horizontally; and the trade-off between competitiveness and risk can be obtained by moving vertically.

Figure 3
Investment Policy Imposed and Varying Spread Allowed

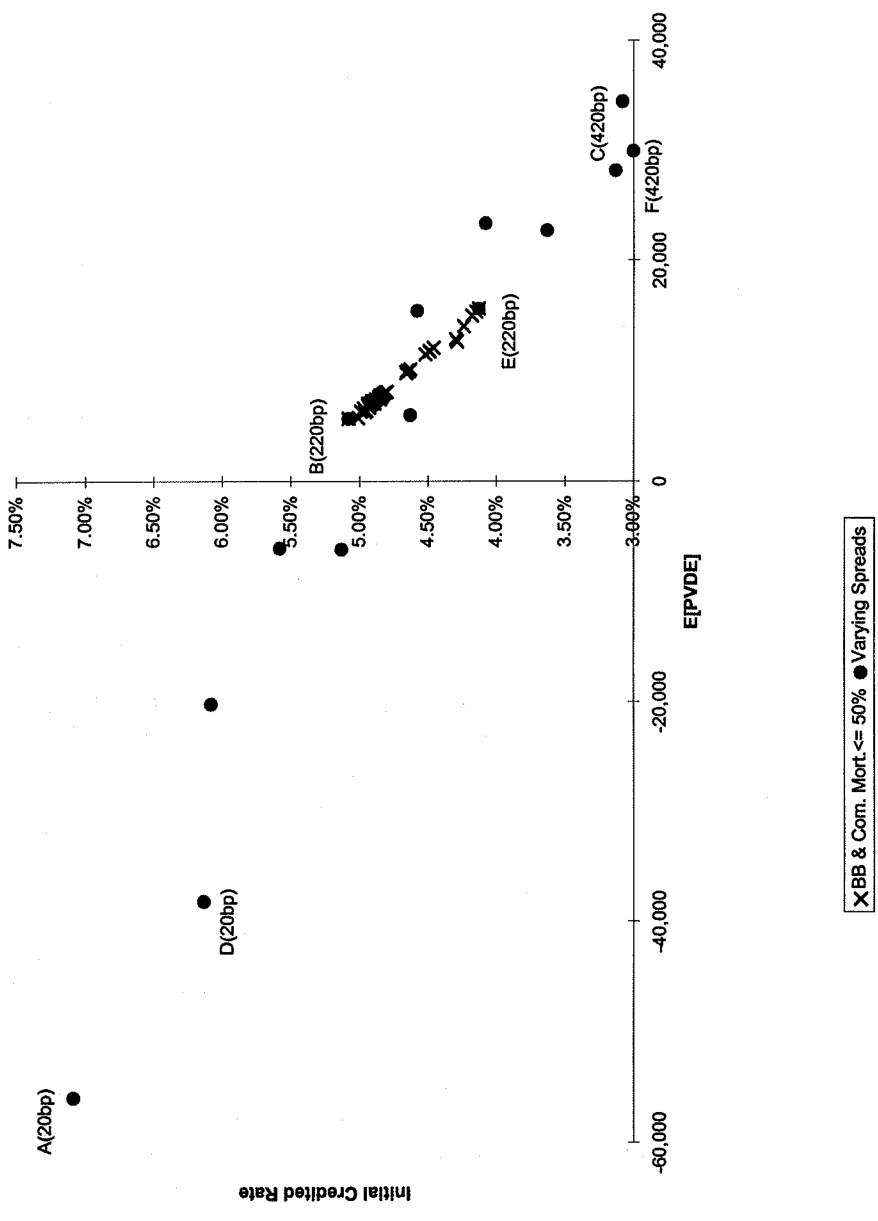
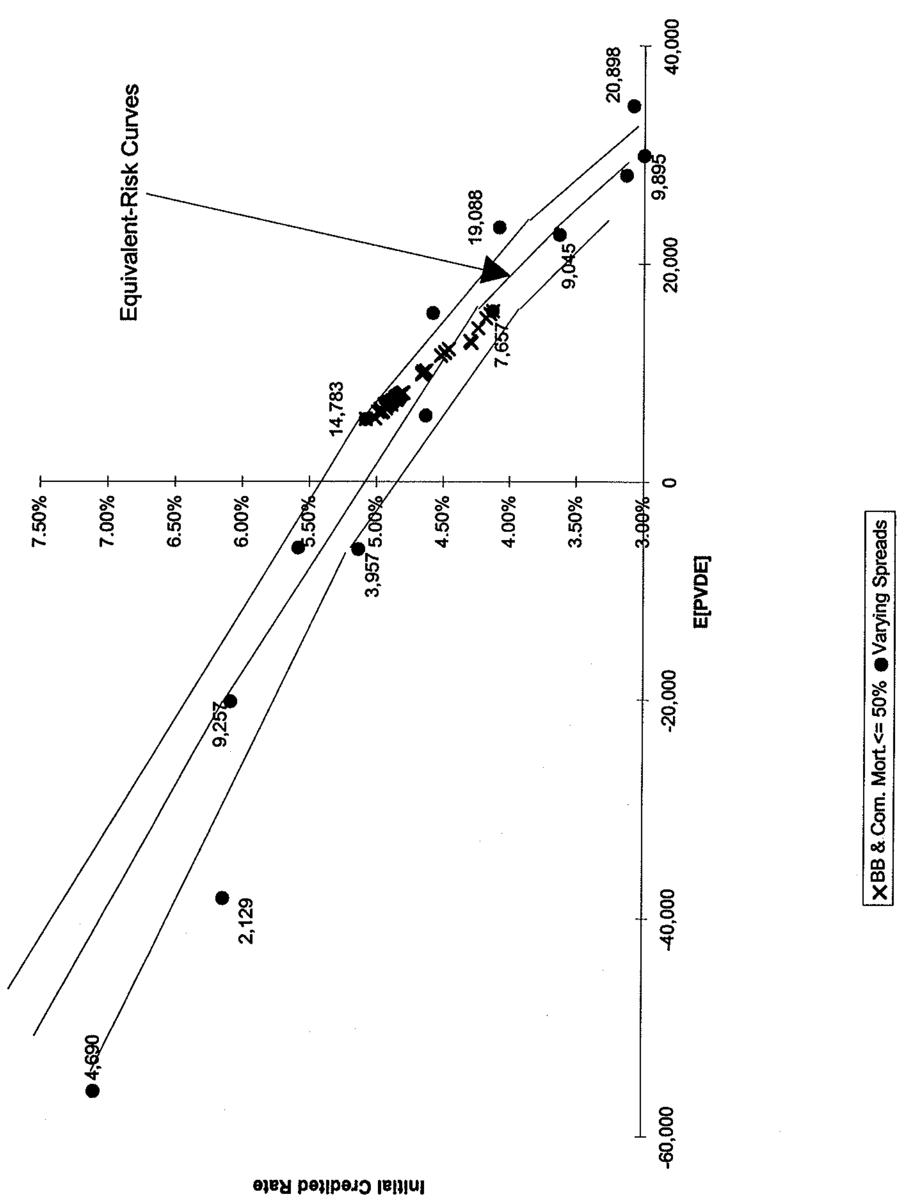


Figure 4
Construction of Equivalent Risk Curves



In Section 1, we introduced four concerns of product development actuaries when doing their pricing work. We can use Figure 4 to address these concerns if we recognize that the first three concerns can be restated as constraints on the three measures we have defined:

- Setting $E[PVDE]$ greater than zero will ensure that the insurance company earn more than its cost of capital;
- Setting $STD[PVDE]$ less than, say, 10,000 (this is just an example, as the actual level will be subjective) will ensure that the risk level is acceptable; and
- Setting the initial credited rate greater than, say, 5.00 percent (again, this is just an example) will make the product sufficiently competitive.

Once these constraints are placed on Figure 4, it will not be difficult to determine the set of asset allocation/spread combinations that will meet these constraints.

It is possible that when the constraints are placed on Figure 4, no possible solution will be found because the area of interest falls outside the area ABCFED. When this occurs, either the constraints must be adjusted to be more realistic or more asset types must be included in the analysis to expand the solution set to an area larger than ABCFED. The search for new asset classes is currently under way at many insurance companies as the competition they face intensifies. Some of these new asset classes include asset-backed securities, credit derivatives, and emerging market debts. If the constraints cannot be adjusted and no new asset classes can be found, the company may want to reconsider the feasibility of selling the product.

A new asset class may be superior to existing asset classes because it is able to take advantage of loopholes in the risk-based capital formula. For example, an asset-backed security may be given an investment grade rating by the NAIC when the assets backing the security are noninvestment grade assets. Thus, the insurance company buying the security will gain from the higher yield without having to put in the extra capital. Such loopholes will not last forever. Once regulators become aware of the loopholes, they will take measures to correct the situation.

Some insurance companies may try to increase profitability by reducing the amount of capital they use to fund their products. For example, they may decide to change the target surplus they use from 200 percent to 150 percent of the NAIC company action level capital.

Such actions eventually will result in downgrades of their ratings. When downgrades occur the cost of capital will increase, and profitability will decrease on a present value basis. Also, downgraded ratings will make their products more difficult to sell. Maintaining the competitiveness of their products will require crediting higher rates, again reducing profitability toward the original level.

5 Summary and Conclusion

This paper presents a flexible methodology that meets the needs of product development actuaries. The methodology can be adapted to any product as long as the distributable earnings for the product can be modeled. Other asset types, including derivatives, can be added as long as the asset cash flows can be modeled. The weighting of the various components of the fitness score also can be adjusted to emphasize one of the three measures.

The methodology is superior to the traditional pricing model in three ways.

- The use of multiple scenarios allows the assessment of the risk of the product. Under the traditional approach where a static economic environment is assumed, the risk of not achieving the required profit objectives due to changing economic environments cannot be measured.
- The use of cost of capital that changes with the yield curve results in a more accurate profit measure. Under the traditional approach, distributable earnings are discounted at a fixed percentage. This ignores the fact that earning 12 percent when the 90 day treasury rate is 5 percent is different from earning 12 percent when the 90 day treasury rate is 15 percent.
- The use of an asset-liability model allows us to determine the appropriate asset allocation to back the product. Under the traditional approach, assets are not modeled. As such, the product development actuaries are not involved in the selection of the asset allocation.

The investment community has long been aware of the first two weaknesses of the traditional approach. This is evident in the pricing of mortgage-backed securities by Hayre and Lauterbach (1995) where

Monte Carlo simulation techniques are used to overcome both the problem of path-dependent cash flows and the problem of discounting cash flows at a fixed rate under different paths.

Asset-liability management actuaries have used the same techniques to value liability cash flows and to calculate their durations and convexities. These models do not serve product development actuaries well because little emphasis is placed on profitability and the competitiveness of the product. This paper combines the strengths of both worlds and presents a general framework that can be adapted to any product.

Despite these strengths, areas exist where further research is needed. Using the simple genetic algorithm described in this paper to seek the profitability-risk-competitiveness frontier is found to be efficient. Genetic algorithms different from the one described in this paper, however, may be more efficient especially since the genetic algorithm we use lacks randomness and mutations.

With the advent of more complicated products such as equity index annuities and life insurance, the need for models to simultaneously generate both equity and interest rate scenarios is compelling. Also, studies on policyholder behavior under various equity/interest rate environments are needed to model the product more accurately.

CAPM is used to find the risk premium in equation (1). With all the controversy surrounding the appropriateness of CAPM, more work is needed to find a better method to calculate the risk premium. For mortgage-backed securities where a broad market exists, the market values can be used to back into the correct option-adjusted spread. Without a market for insurance products, it will be a challenge to create an accurate measure of the risk premium.

References

American Academy of Actuaries Task Force on Fair Valuation of Liabilities. *Fair Valuation of Life Insurance Company Liabilities.* Washington, D.C.: American Academy of Actuaries, October 30, 1995.

Asay, M.R., Bouyoucos, P.J. and Marciano, A.M. "An Economic Approach to Valuation of Single Premium Deferred Annuities." In S.A. Zenios (ed.), *Financial Optimization.* New York, N.Y.: Cambridge University Press, 1993: 101–135.

Barber, J.C. "Genetic Algorithms as Tools for Optimization." Society of Actuaries Newsletter *Risks and Rewards* 24 (December 1995): 1, 5–6.

Brealey, R.A. and Myers, S.C. *Principles of Corporate Finance.* McGraw-Hill, Inc., 1991.

Brigger, P. *Morphological Shape Representation Using the Skeleton Decomposition: Application to Image Coding.* (Ph.D. Dissertation). Lausanne, Switzerland: Ecole Polytechnique Fédérale de Lausanne, 1995.

Goldberg, D.E. *Genetic Algorithms in Search, Optimization, and Machine Learning.* Reading, Mass.: Addison-Wesley, 1989.

Hayre, L.S. and Lauterbach, K. "Option-Adjusted Spread Analysis." In F.J. Fabozzi and T.D. Fabozzi (eds.), *The Handbook of Fixed Income Securities.* Burr Ridge, Ill.: Irwin Professional Publishing, 1995: 635-664.

Holland, J.H. *Adaptation in Natural and Artificial Systems.* Ann Arbor, Mich.: University of Michigan Press, 1975.

Hull, J.C. *Options, Futures, and Other Derivative Securities.* Englewood Cliffs, N.J.: Prentice Hall, 1993.

Kalos, M.H. and Whitlock, P.A. *Monte Carlo Methods, Volume 1: Basics* New York, N.Y.: Wiley, 1986.

Merfeld, T.J. "An Approach to Estimating Market Value and Duration of Interest-Sensitive Whole Life Contracts." *Journal of Actuarial Practice* 4, no. 1 (1996): 5-25.

Reitano, R.R. "Two Paradigms For the Market Value of Liabilities." *North American Actuarial Journal* 1, no. 4 (1997): 104-137.

Wendt, R.Q. "Build Your Own GA Efficient Frontier." Society of Actuaries Newsletter *Risks and Rewards* 24 (December 1995): 1, 4-5.

Table A1
Solution Set After Generation Six

Asset Types*										
3 Year A Bond	5 Year A Bond	7 Year A Bond	10 Year A Bond	10 Year BB Bond	5 Year Comm Mortgage	7 Year Comm Mortgage	10 Year Comm Mortgage	E[PVDE]	STD[PVDE]	Initial Credited Rate
100.00%								15,654	7,657	4.13%
				100.00%				-21,132	22,415	6.04%
					100.00%			-33	8,765	5.48%
						100.00%		-1,496	12,797	5.67%
							100.00%	-1,074	17,445	5.81%
50.00%	50.00%							14,046	10,264	4.24%
50.00%					50.00%			8,098	7,520	4.80%
50.00%							50.00%	6,605	11,884	4.97%
	50.00%					50.00%		5,749	12,718	5.01%
	50.00%						50.00%	5,695	14,783	5.08%
				50.00%	50.00%			-11,072	14,785	5.76%
				50.00%		50.00%		-11,531	17,157	5.86%
				50.00%			50.00%	-11,344	19,760	5.93%
					50.00%	50.00%		-423	10,483	5.57%
					50.00%		50.00%	-121	12,798	5.65%
						50.00%	50.00%	-1,096	14,953	5.74%
75.00%	25.00%							14,993	8,889	4.18%
50.00%	25.00%	25.00%						12,690	11,593	4.29%
50.00%					25.00%	25.00%		7,442	8,507	4.85%
50.00%					25.00%		25.00%	7,444	9,572	4.88%

*No solutions involving five year BB bonds or seven year BB bonds remain after six generations

Table A2
Solution Set After Generation Six

Asset Types*										
3 Year A Bond	5 Year A Bond	7 Year A Bond	10 Year A Bond	10 Year BB Bond	5 Year Comm Mortgage	7 Year Comm Mortgage	10 Year Comm Mortgage	E[PVDE]	STD[PVDE]	Initial Credited Rate
50.00%						25.00%	25.00%	6,658	10,747	4.93%
25.00%	75.00%							12,878	11,629	4.30%
25.00%	25.00%				25.00%	25.00%		7,101	10,008	4.91%
25.00%	25.00%				25.00%		25.00%	7,063	11,066	4.94%
25.00%	25.00%					50.00%		6,389	10,964	4.95%
25.00%	25.00%					25.00%	25.00%	6,345	11,962	4.99%
	50.00%				25.00%	25.00%		6,317	11,814	4.96%
		25.00%			50.00%		25.00%	2,746	13,194	5.33%
		25.00%			25.00%	25.00%	25.00%	2,181	14,202	5.38%
		25.00%			25.00%		50.00%	2,222	15,268	5.41%
			25.00%		75.00%			1,938	12,822	5.31%
			25.00%		50.00%	25.00%		1,683	13,708	5.35%
			25.00%		50.00%		25.00%	1,724	14,929	5.39%
				50.00%	25.00%	25.00%		-11,081	15,812	5.81%
				25.00%	25.00%	50.00%		-6,029	13,574	5.71%
				25.00%	25.00%	25.00%	25.00%	-5,831	14,728	5.75%
				25.00%	25.00%		50.00%	-5,779	16,005	5.79%
				25.00%		75.00%		-6,578	14,864	5.76%
				25.00%		50.00%	25.00%	-6,371	16,019	5.80%
				25.00%		25.00%	50.00%	-6,251	17,236	5.83%

*No solutions involving five year BB bonds or seven year BB bonds remain after six generations

Table A3
Solution Set After Generation Six

Asset Types*										
3 Year A Bond	5 Year A Bond	7 Year A Bond	10 Year A Bond	10 Year BB Bond	5 Year Comm Mortgage	7 Year Comm Mortgage	10 Year Comm Mortgage	E[PVDE]	STD[PVDE]	Initial Credited Rate
					75.00%	25.00%		-123	9,532	5.53%
					75.00%		25.00%	69	10,745	5.56%
					50.00%	25.00%	25.00%	-230	11,582	5.61%
						75.00%	25.00%	-1,227	13,850	5.70%
						25.00%	75.00%	-1,036	16,132	5.78%
87.50%	12.50%							15,372	8,242	4.15%
75.00%					25.00%			12,114	7,552	4.46%
75.00%					12.50%	12.50%		11,762	8,064	4.49%
62.50%	12.50%				25.00%			11,849	8,234	4.49%
62.50%	12.50%				12.50%	12.50%		11,518	8,728	4.52%
50.00%					37.50%	12.50%		7,800	8,004	4.83%
50.00%					12.50%	25.00%	12.50%	7,034	9,545	4.89%
37.50%	12.50%				50.00%			7,913	8,290	4.83%
37.50%	12.50%				25.00%	12.50%	12.50%	7,324	9,760	4.89%
37.50%	12.50%				25.00%		25.00%	7,297	10,299	4.91%
37.50%	12.50%				12.50%	25.00%	12.50%	6,939	10,246	4.92%
37.50%	12.50%				12.50%	12.50%	25.00%	6,898	10,753	4.94%
25.00%	25.00%				25.00%	12.50%	12.50%	7,098	10,525	4.92%
		12.50%	12.50%		62.50%		12.50%	2,422	12,981	5.32%
		12.50%			75.00%		12.50%	1,681	10,904	5.41%

*No solutions involving five year BB bonds or seven year BB bonds remain after six generations

Table A4
Solution Set After Generation Six

Asset Types*										
3 Year A Bond	5 Year A Bond	7 Year A Bond	10 Year A Bond	10 Year BB Bond	5 Year Comm Mortgage	7 Year Comm Mortgage	10 Year Comm Mortgage	E[PVDE]	STD[PVDE]	Initial Credited Rate
		12.50%			62.50%	12.50%	12.50%	1,509	11,359	5.43%
		12.50%			62.50%		25.00%	1,555	11,916	5.45%
		12.50%			50.00%	25.00%	12.50%	1,302	11,839	5.45%
		12.50%			50.00%	12.50%	25.00%	1,361	12,385	5.47%
			12.50%		87.50%			1,137	10,769	5.40%
			12.50%		75.00%	12.50%		1,083	11,167	5.42%
			12.50%		75.00%		12.50%	1,135	11,805	5.44%
			12.50%		62.50%	25.00%		954	11,615	5.44%
			12.50%		62.50%	12.50%	12.50%	1,016	12,202	5.46%
				12.50%	12.50%	62.50%	12.50%	-3,596	13,663	5.71%
				12.50%	12.50%	50.00%	25.00%	-3,501	14,214	5.73%
				12.50%	12.50%	37.50%	37.50%	-3,429	14,782	5.75%
				12.50%	12.50%	25.00%	50.00%	-3,383	15,369	5.76%
					87.50%	12.50%		-40	9,115	5.51%
					87.50%		12.50%	79	9,747	5.52%
					75.00%	12.50%	12.50%	-17	10,109	5.55%
					62.50%	37.50%		-257	9,992	5.55%
					62.50%	25.00%	12.50%	-146	10,549	5.57%
					62.50%	12.50%	25.00%	-77	11,121	5.59%
					50.00%	37.50%	12.50%	-311	11,027	5.59%

*No solutions involving five year BB bonds or seven year BB bonds remain after six generations

Table A5
Solution Set After Generation Six

Asset Types*										
3 Year A Bond	5 Year A Bond	7 Year A Bond	10 Year A Bond	10 Year BB Bond	5 Year Comm Mortgage	7 Year Comm Mortgage	10 Year Comm Mortgage	E[PVDE]	STD[PVDE]	Initial Credited Rate
						62.50%	37.50%	-1,153	14,393	5.72%
						37.50%	62.50%	-1,056	15,532	5.76%
						12.50%	87.50%	-1,052	16,755	5.79%
62.50%					37.50%			10,149	7,516	4.63%
62.50%					31.25%	6.25%		9,986	7,759	4.64%
62.50%					25.00%	12.50%		9,816	8,011	4.66%
56.25%	6.25%				37.50%			10,043	7,870	4.65%
56.25%	6.25%				31.25%	6.25%		9,884	8,109	4.66%
50.00%	12.50%				37.50%			9,914	8,234	4.66%
50.00%					31.25%	18.75%		7,627	8,252	4.84%
43.75%	6.25%				50.00%			8,017	7,899	4.82%
43.75%	6.25%				43.75%	6.25%		7,880	8,132	4.83%
43.75%	6.25%				37.50%	12.50%		7,738	8,377	4.84%
43.75%	6.25%				31.25%	18.75%		7,573	8,622	4.85%
37.50%	12.50%				43.75%	6.25%		7,786	8,520	4.84%
37.50%			6.25%		56.25%			7,002	7,947	4.90%
31.25%	18.75%				50.00%			7,782	8,690	4.85%
31.25%	18.75%				43.75%	6.25%		7,666	8,914	4.86%
25.00%			6.25%		68.75%			4,497	8,845	5.10%
25.00%			6.25%		62.50%	6.25%		4,412	9,065	5.11%

*No solutions involving five year BB bonds or seven year BB bonds remain after six generations

Table A6
Solution Set After Generation Six

Asset Types*										
3 Year A Bond	5 Year A Bond	7 Year A Bond	10 Year A Bond	10 Year BB Bond	5 Year Comm Mortgage	7 Year Comm Mortgage	10 Year Comm Mortgage	E[PVDE]	STD[PVDE]	Initial Credited Rate
18.75%	6.25%		6.25%		68.75%			4,413	9,286	5.11%
		6.25%			87.50%		6.25%	926	9,800	5.44%
		6.25%			81.25%	6.25%	6.25%	883	10,002	5.46%
		6.25%			81.25%		12.50%	924	10,301	5.46%
		6.25%			75.00%	12.50%	6.25%	821	10,217	5.47%
		6.25%			75.00%	6.25%	12.50%	863	10,506	5.48%
		6.25%			68.75%	18.75%	6.25%	748	10,443	5.48%
		6.25%			68.75%	12.50%	12.50%	791	10,724	5.49%
		6.25%			62.50%	25.00%	6.25%	665	10,675	5.49%
				6.25%	56.25%	31.25%	6.25%	-1,650	11,008	5.60%
				6.25%	50.00%	37.50%	6.25%	-1,724	11,253	5.61%
				6.25%	50.00%	31.25%	12.50%	-1,674	11,532	5.62%
				6.25%	43.75%	43.75%	6.25%	-1,806	11,503	5.62%
				6.25%	43.75%	37.50%	12.50%	-1,753	11,779	5.63%
				6.25%	37.50%	50.00%	6.25%	-1,895	11,757	5.63%
				6.25%	37.50%	43.75%	12.50%	-1,841	12,031	5.64%
				6.25%	31.25%	56.25%	6.25%	-1,993	12,019	5.64%
					93.75%	6.25%		-35	8,925	5.49%
					93.75%		6.25%	40	9,248	5.50%
					87.50%	6.25%	6.25%	13	9,415	5.51%

*No solutions involving five year BB bonds or seven year BB bonds remain after six generations

Table A7
Solution Set After Generation Six

Asset Types*										
3 Year A Bond	5 Year A Bond	7 Year A Bond	10 Year A Bond	10 Year BB Bond	5 Year Comm Mortgage	7 Year Comm Mortgage	10 Year Comm Mortgage	E[PVDE]	STD[PVDE]	Initial Credited Rate
					81.25%	18.75%		-74	9,316	5.52%
					81.25%	12.50%	6.25%	-15	9,610	5.53%
					81.25%	6.25%	12.50%	27	9,912	5.54%
					75.00%	18.75%	6.25%	-66	9,814	5.54%
					68.75%	31.25%		-185	9,759	5.54%
					68.75%	25.00%	6.25%	-125	10,038	5.55%
					68.75%	18.75%	12.50%	-77	10,321	5.56%
					62.50%	31.25%	6.25%	-196	10,269	5.56%
					56.25%	43.75%		-337	10,234	5.56%
					56.25%	37.50%	6.25%	-276	10,509	5.57%
46.88%	3.13%				50.00%			8,061	7,709	4.81%
43.75%			3.13%		53.13%			7,265	8,072	4.87%
25.00%		3.13%			68.75%		3.13%	4,390	8,371	5.12%
25.00%					75.00%			4,002	7,804	5.14%
25.00%					71.88%	3.13%		3,955	7,902	5.15%
25.00%					65.63%	9.38%		3,887	8,127	5.16%
21.88%	3.13%	3.13%			68.75%		3.13%	4,364	8,588	5.13%
21.88%	3.13%				75.00%			3,985	8,028	5.15%
21.88%	3.13%				71.88%		3.13%	3,966	8,278	5.16%
21.88%	3.13%				68.75%	6.25%		3,913	8,232	5.16%

*No solutions involving five year BB bonds or seven year BB bonds remain after six generations

Table A8
Solution Set After Generation Six

Asset Types*										
3 Year A Bond	5 Year A Bond	7 Year A Bond	10 Year A Bond	10 Year BB Bond	5 Year Comm Mortgage	7 Year Comm Mortgage	10 Year Comm Mortgage	E[PVDE]	STD[PVDE]	Initial Credited Rate
21.88%	3.13%				65.63%	9.38%		3,876	8,343	5.17%
18.75%		3.13%	3.13%		71.88%		3.13%	3,663	9,027	5.19%
18.75%			3.13%	3.13%	56.25%	15.63%	3.13%	2,286	9,619	5.26%
18.75%			3.13%		78.13%			3,281	8,466	5.21%
18.75%			3.13%		75.00%		3.13%	3,265	8,720	5.22%
18.75%			3.13%		71.88%	6.25%		3,220	8,669	5.22%
		3.13%			93.75%		3.13%	459	9,266	5.46%
		3.13%			90.63%	3.13%	3.13%	462	9,362	5.47%
		3.13%			90.63%		6.25%	480	9,510	5.47%
		3.13%			84.38%	9.38%	3.13%	438	9,557	5.48%
				3.13%	68.75%	25.00%	3.13%	-852	10,147	5.56%
					96.88%	3.13%		-48	8,834	5.49%
					96.88%		3.13%	8	9,005	5.49%
					93.75%	3.13%	3.13%	-15	9,076	5.50%
					90.63%	9.38%		-32	9,020	5.50%
					90.63%	6.25%	3.13%	-8	9,169	5.50%
					87.50%	9.38%	3.13%	-9	9,265	5.51%

*No solutions involving five year BB bonds or seven year BB bonds remain after six generations